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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, August 2021

First Degree Programme Under CBCSS

Statistics

Core Course

ST 1341 : PROBABILITY AND DISTRIBUTION I

(2019 Admission Regular)

Re Examination

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sample space of a random experiment.

2. What is the probability that a non-leap year will contain 53 Sundays?

3. Define the distribution function of a random variable.

4. Write the properties of a probability mass function.

5. Explain the bivariate random variable.

6. Define independence of two random variables.

P.T.O.

7. State the multiplication theorem on expectation.

8. Define conditional variance of a random variable.

9. For what type of random variables do the probability generating function exist?

10. Give an example of a random variable whose moment generating function does not exist.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Give the mathematical/classical definition of probability. What are its limitations?

12. Let A and B be two events which are not mutually exclusive and if $P(A)=\frac{1}{4}, P(B)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$. Then find the probability that (i) either one of them will happen and (ii) none of them will happen.

13. Distinguish between mutually exclusive events and independent events.

14. Let A and B be two events which are not mutually exclusive and if $P(A)=\frac{1}{4}, P(B)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$. Find the probabilities of A given B and also B given A.

15. If X has *pmf* $f(x)\frac{1}{3}, x=1,2,3$ and 0 elsewhere. Find the pmf of $Y=2X+1$.

16. Let $f(x)=|x|$ for $|x|\leq c$ and $f(x)=0$ elsewhere. Find the value of c, so that $f(x)$ is a pdf.

17. A coin is tossed with a probability p to get a head in a single trial. Write $P[X = x]$, if X denotes the number of trials required to get a head.

18. Define the joint distribution of a pair of continuous random variables.

19. If $f(x / y) = \frac{cx}{y^2}, 0 \leq x < y \leq 1$ is a conditional pdf, find c?

20. Find the distribution of the distribution function $F(x)$ of the random variable X.

21. State the Cauchy-Schwartz inequality.

22. Two unbiased dice are thrown. Find the expectation of the sum of the number of points on them.

23. Define the coefficient of linear correlation between two random variables.

24. What is the effect of change of origin and scale in moment generating function?

25. How do you get the r^{th} non-central (row) moment from the characteristic function $\phi_X(t)$?

26. Define the cumulant generating function of a random variable.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Give the axiomatic definition of probability.

28. State and prove the addition law of probability.

29. The probability that a man will be alive for 25 years is 3/5 and that his wife will be alive for 25 years s 2/3. Find the probability that (i) both will be alive, (ii) only wife will be alive. (iii) at least one of them will be alive and (iv) none of them will be alive

30. If X has the pdf $f(x)=\frac{1}{2}, -1<x<1$ and 0 elsewhere, what is the pdf of $Y=X^2$?

31. Explain the technique of transformation of random variables.

32. A continuous r.v X has the pdf $f(x)=2x, 0<x\leq 1$, and 0 elsewhere. Find (1) $F(x)$, (2) $P\left(X\leq \frac{1}{2}\right)$ & (3) $P\left(\frac{1}{4}\leq X\leq \frac{3}{4}\right)$.

33. If (X,Y) has the joint pdf $f(x,y)=\frac{xy^2}{30}, x=1,2,3$ & $y=1,2$. Find the pdfs of X and Y and examine whether X and Y are independent or not.

34. The joint pdf of the random vector (X,Y) is $f(x,y)=2, 0<x<y<1$ and 0 otherwise. Find the marginal and conditional pdfs of X and Y

35. Prove that $E(E(X/Y))=E(X)$

36. Show that $E(X^2)\geq (E(X))^2$

37. Mention the important properties of a characteristic function.

38. Show that $M_{X+Y}(t)=M_X(t)M_Y(t)$, where X and Y are independent random variables.

(6 x 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) State and prove the Baye's Theorem.

(b) In a bolt factory, machines M_1, M_2 and M_3 manufacture respectively 25,35 and 40 percent of the total output. Of their outputs 5,4 and 2 percent respectively are defective bolts. One bolt taken at random from the product and is found to be defective. What is the probability that it is manufactured by machine M_2?

40. X is a discrete random variable having the following probability distribution.

X	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k, (ii) $P(X<6)$, (iii) $P(X \geq 6)$, (iv) $P(X>6)$, and (v) $P(0-<X<5)$.

41. For the joint pdf $f(x,y)=x+y, 0<x<1, 0<y<1$, obtain the marginal pdfs of X and Y and also $P\left(X>\frac{1}{2} / Y>\frac{1}{2}\right)$.

42. Let $f(x,y)=\frac{1}{72}(2x+3y), x=0,1,2 \;\&\; y=1,2,3$ be the joint pdf of (X,Y). Then find (i) the distribution of $X+Y$, (ii) the conditional distribution of $X/X+Y=3$ and (iii) examine whether X and Y are independent.

43. Let the joint pdf of a discrete random vector (X, Y) be given in the table.

X / Y	0	1	2	3
0	0	1/8	1/4	1/8
1	1/8	1/4	1/8	0

Find $E(X / Y = 2)$, $E(Y / X = 1)$, $V(X / Y = 2)$ and $V(Y / X = 1)$.

44. Find the moment generating function of X with pdf $f(x) = \frac{1}{\theta}, 0 < x < \theta$ and 0 elsewhere. Also compute the mean, variance and the first 4 central moments.

(2 × 15 = 30 Marks)
