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E – 3389

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2018
First Degree Programme under CBCSS
Mathematics
Core Course – III
MM 1441 : ALGEBRA AND CALCULUS – II
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Show that $(1 + 2x)$ is a unit of $R[x]$ where $R = \mathbb{Z}/4\mathbb{Z}$.

2. Find the remainder in $Q[x]$ when $x^4 - 7x^2 + 3$ is divided by $(x + 1)$.

3. State primitive element theorem.

4. State true or false : $x^3 - 2$ is irreducible in $Q[x]$. Justify your answer.

5. Write an irreducible polynomial in $F_2[x]$ of degree 4.

6. If $f(x, y) = x + \sqrt[3]{xy}$, find $f(t, t^2)$.

7. Show that $\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}$ does not exist.

8. Find the natural domain of $f(x, y) = \ln(x^2 - y)$.

9. Evaluate $\int_0^{\ln 2}\int_0^{\ln 2} e^{x+y}\, dy dx$.

10. Find a parametric representation of $z = 4 - x^2 - y^2$.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Detach the coefficients and find $(x^3 + 1)(2x^2 + 2)$ in $F_3[x]$.

12. State fundamental theorem of algebra.

13. Using Euclid's algorithm, find the gcd of $(x^3 + 1)$ and $(x^3 + x + 1)$ in $F_3[x]$.

14. For n = 8, verify $\Sigma_{d|n} \phi(d) = n$.

15. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^4 \sin(xy)$.

16. If $f(x) = x^2 + bx + c$ is a polynomial of degree 2 in $R[x]$, then prove that f(x) is irreducible if $b^2 - 4c < 0$.

17. Find the slope of the surface $z = \sqrt{(3x + 2y)}$ in the x-direction at the point (4, 2).

18. Evaluate $\int_0^2 \int_{x^2}^{x} y^2 x \, dy \, dx$.

19. Find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $[0, 1] \times [0, 2]$.

20. If $f(x, y) = x^2y^3 + x^4y$, find f_{xyy}.

21. Evaluate $\int_0^2 \int_{-1}^{y} \int_{-1}^{z} yz \, dx \, dz \, dy$.

22. Factor $x^3 - x$ into irreducible polynomials in $Z/_{3Z}[x]$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Find the quotient and remainder when the polynomial $x^4 - 2x^2 - 1$ is divided by $x^2 + 3x - 1$.

24. Find the gcd d(x) in Q[x] of f(x) and g(x) and find polynomials r(x) and s(x) with $f(x) r(x) + g(x) s(x) = d(x)$ where $f(x) = x^2 - 3x + 2$ and $g(x) = x^2 + x + 1$.

25. For any $e > 1$ dividing $p - 1$, if $N(e) > 0$, then $N(e) = \emptyset(e)$. State true or false. Justify your answer

26. Write $x^2 + 3x + 1$ in base $(x + 2)$.

27. Given that $z = e^{xy}$, $x = 2u + v$, $y = u/v$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

28. Show that the function $Z = \ln(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ satisfies Laplace's equation.

29. Locate all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

30. Find the area of the cardiod $r = 2(1+\cos\theta)$.

31. Evaluate $\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$ by reversing the order of integration.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) State and prove division theorem.

 b) Prove that if $f(x)$ is a polynomial with coefficient in a field F and a is in F, then $f(a) = 0$ if and only if $x-a$ divides $f(x)$.

33. Find a solution of $x^3 + 6x + 8 = 6x^2$ by Cardano's method.

34. a) Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top having a volume of 32 ft^3 and requiring the least amount of material for its construction.

 b) Let $z = f(x^2 - y^2)$, show that $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$.

35. a) Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

 b) Use triple integral, find volume of the solid bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$.