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Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, July 2018
(First Degree Programme Under CBCSS)
Mathematics
Core Course : III
MM 1441 : METHODS OF ALGEBRA AND CALCULUS II
(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. Detach the coefficients and multiply $(x^3 + 1)(2x^2 + 2)$ in $\mathbb{F}_3[x]$.
2. Show that $2x + 1$ is a unit of the ring $R[x]$; $R = \mathbb{Z}/4\mathbb{Z} = \{0, 1, 2, 3\}$.
3. Find the remainder in $\mathbb{Q}[x]$ when $x^3 - 2x + 4$ is divided by $x - 2$.
4. Define an irreducible polynomial over a field F.
5. State the Fundamental theorem of Algebra.
6. Find f(e, 0) if $f(x, y) = \sqrt{y+1} + \ln(x^2 - 1)$.
7. Find $\lim_{(x,y)\to(3,4)} \sqrt{x^2 + y^2 - 1}$.
8. Find $\frac{\partial f}{\partial x}$ at (4, –5) if $f = x^2 + 3xy + y - 1$.
9. Find the parametric equation of the right circular cylinder $x^2 + y^2 = 9$ for which $0 \le y \le 5$.
10. Find the point at which $f(x, y) = 3xy - 6x - 3y + 7$ has an absolute minimum.

P.T.O.

SECTION – II

Answer **any 8** questions among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Find a polynomial q(x) in $\mathbb{Z}/2\mathbb{Z}$ which is equal to p(x) = x + 1. Also prove that p(x) = q(x).

12. For which values of k in $\mathbb{Q}$ does $x - k$ divide $x^3 - kx^2 - 2x + k + 3$?

13. For any $e \geq 1$, prove that $x^e - 1$ has at most e roots in $\mathbb{Z}/p\mathbb{Z}$.

14. Is the polynomial $x^2 + 1$ irreducible in $\mathbb{R}[x]$?

15. Find all irreducible polynomials of degree 2 in $\mathbb{Z}/2\mathbb{Z}$.

16. Decompose into partial fractions $\frac{t+1}{(t-1)(t+2)}$.

17. Find the slope of the surface z = f(x, y) in the x – direction at the point (1, – 2) where $f(x, y) = x^2y + 5y^3$.

18. If $w = x^2y - y^2$; $x = \sin t$, $y = e^t$, find $\frac{dw}{dt}$ at t = 0.

19. Find the value of $\int_0^2 \int_0^1 (4 - x - y)\,dx\,dy$.

20. Use the implicit differentiation to find $\frac{\partial y}{\partial x}$ if $y^3 + y^2 - 5y - x^2 + 4 = 0$.

21. Find the partial derivatives of the vector valued function $\bar{r} = ui + vj + \left(4 - u^2 - v^2\right)k$.

22. Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

SECTION – III

Answer **any 6** questions among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Solve $x^3 + 3x = 14$.

24. Use Newton's method to find a real root of the equation $x^3 - x - 1 = 0$.

25. Prove that if R is a commutative ring, then so is R[x].

26. State and prove the Remainder theorem.

27. Prove that $N(p-1) = \phi(p-1)$.

28. Let $f(x,y) = \begin{cases} \frac{-xy}{x^2+y^2}; & (x,y) \neq 0 \\ 0 \quad ; & (x,y) = 0 \end{cases}$

    a) Show that $f_x(x, y)$ and $f_y(x, y)$ exist at all points (x, y).

    b) Explain why f is not continuous at (0, 0).

29. Locate all relative extrema and find the relative extreme values of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

30. Evaluate $\iint_R xy\, dA$ over the region R enclosed between $y = \frac{1}{2}x$, $y = \sqrt{x}$, $x = 2$ and $x = 4$.

31. Evaluate the triple integral $\iiint_G 12xy^2z^3\, dV$ over the rectangular box G defined by $-1 \leq x \leq 2$, $0 \leq y \leq 3$, $0 \leq z \leq 2$.

SECTION – IV

Answer **any 2** questions among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Find all roots of the equation $x^3 = 7x + 6$ using Cardano's method.

    b) Find all roots in $\mathbb{Z}/15\mathbb{Z}$ of $f(x) = x^2 - 2x$.

33. a) Prove that a non zero polynomial f(x) of degree n in F[x]; F a field, has at most n distinct roots in F.

    b) Prove that any polynomial of degree greater than or equal to 1 in F[x]; F a field, is irreducible or factors into a product of irreducible polynomials.

34. a) Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0, 5).

    b) Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter p and maximum area.

35. a) Evaluate $\iint_R (2x - y^2)\, dA$ over the rectangular region R enclosed between the lines $y = -x + 1$, $y = x + 1$ and $y = 3$.

    b) Use the triple integral to find the volume of the solid within the Cylinder $x^2 + y^2 = 9$ and between the lines $z = 1$ and $x + z = 5$.