

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Economics

MM 1131.5 : MATHEMATICS FOR ECONOMICS I

(2013-2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION - I

Answer all questions. These questions carry 1 mark each.

1. Find the natural domain of $f(x) = \sqrt{x+1} + 4$.2. Evaluate $\lim_{x \rightarrow \frac{1}{2}} \frac{x^n - \left(\frac{1}{2}\right)^n}{x - \frac{1}{2}}$.3. Given the cost function, $\pi = 100 + 8x + 0.1x^2$, find out the marginal cost when $x = 50$.4. Find $\frac{dy}{dx}$ at (1, 1) when $x + y + xy = 3$.5. For a linear cost function $TC = a + bq$, show that the marginal cost is a constant.

6. Find the slope of the tangent line to the curve $y = x^3 - x$ at $x = 2$.
7. Give an example of a function that is continuous but not differential at a point.
8. Obtain a relation between the variables x and y if it is given that $x = t^2$ and $y = 2t$.
9. Write down the derivative of $\log_a x$.
10. What is the value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$?

(10 × 1 = 10 Marks)

SECTION – II

Answer any eight questions. These questions carry 2 marks each.

11. Find the formula for the inverse of $f(x) = \sqrt{3x - 2}$.
12. Draw the graph of $xy = 5$.
13. For the demand function $x = \frac{90}{p+5} - 6$, at what price does the demand tend to vanish?
14. For the demand function $q = 25 - 4p + p^2$, show that the elasticity of demand at $p = 5$ is unitary.
15. Discuss the continuity of the function $f(x) = \sqrt{9 - x^2}$.
16. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{x^2 + 1}$.

17. For what values of x is there a discontinuity in the graph of $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$?
18. If the demand law is given by $p = \frac{a}{x} - c$, show that the total revenue decreases as output increases.
19. Differentiate the function $\frac{x^2 - 1}{x^2 + 1}$ with respect to x .
20. Find the slope of the tangent line to the curve $x + y + xy = 3$ at the point $(1, 1)$.
21. If x and y satisfy the relation $x^2 + y^2 = 1$, show that $\frac{dy}{dx} = -\frac{x}{y}$.
22. If the total cost-function is $\pi = ax^2 + bx + c$, find the marginal cost.

(8 × 2 = 16 Marks)

SECTION - III

Answer any six questions. These question carry 4 marks each.

23. Find the value of k so that $\begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$ is a continuous function.
24. For the curve $q = 36 - 2p$, what can you conclude about the slopes of the MR function and the AR function?

25. If $f(x)$ is a single valued function of x , express the derivatives of $\sqrt{f(x)}$ and reciprocal in terms of the derivative of $f(x)$.
26. Find $\frac{dy}{dx}$ when
- (a) $4x^2 - 2y^2 = 9$
- (b) $y = \frac{1}{\sqrt{x+1}}$
27. If $y = 3x^4 - 2x^3 + x^2 - 4x + 2$, find $\frac{dy}{dx}$.
28. Find the function inverse to $y = \frac{2x+1}{x-1}$ and show that it is single valued. Represent it graphically.
29. Show that the marginal revenue can always be expressed as $p + x \frac{dp}{dx}$. Deduce that the gradient of the demand curve is numerically equal to $\frac{p}{x}$ at the output where marginal revenue is zero.
30. If $f(x) = \frac{4x^2 - x}{2x^3 - 5}$, find $\lim_{x \rightarrow \infty} f(x)$.
31. If $f(x) = 4 + \frac{1}{1 + \frac{1}{1-x}}$, $0 < x < 1$, find $\lim_{x \rightarrow 1} f(x)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These question carry **15** marks each.

32. (a) If $f(x) = x^2$ and $g(x) = \sin x$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- (b) Find the domain and range of $f(x) = \frac{1+x}{1-x}$.
- (c) If $f(x) = \frac{x^2 + 3x - 2}{x^2 + 2x + 4}$, express $f(2a)$ in terms of a .
33. (a) Given that the function $f(x) = \begin{cases} 5-x & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$
- (i) Draw the graph of the function
- (ii) Identify the discontinuity of the function in the graph
- (iii) Find $\lim_{x \rightarrow 4} (5-x)$ and show that the value of the limit is not equal to the value of the function at $x = 4$. What do you conclude?
- (b) From the relation $xy + 2x + y - 1 = 0$, find the limit of y as $x \rightarrow 1$, and the limit of x as $y \rightarrow 1$.
34. (a) Use differential calculus to explain the concept of marginal revenue.
- (b) The demand function of a monopolists is given by $p = 1500 - 2x - x^2$. Find the total revenue, average revenue and marginal revenue functions. Also find the marginal revenue at $x = 20$

35. Find $\frac{dy}{dx}$ if

(a) $y = \frac{e^{2x} \ln x}{x^2}$

(b) $y = \sqrt{\frac{1+x}{1-x}}$

(c) $x^2 + y^2 + 2x + 4y + 5 = 0.$

(2 × 15 = 30 Marks)