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N – 3953

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1131.5 : MATHEMATICS FOR ECONOMICS — I

(2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

(Answer all questions. These questions carry 1 mark each).

1. Find the natural domain of $f(x) = \sqrt{x+1} + 4$.

2. Evaluate $\lim_{x \rightarrow \frac{1}{2}} \frac{x^n - \left(\frac{1}{2}\right)^n}{x - \frac{1}{2}}$.

3. Given the cost function $\pi = 100 + 8x + 0.1x^2$, find out the marginal cost when $x = 50$.

4. Find $\frac{dy}{dx}$ at $(1,1)$ when $x + y + xy = 3$.

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5. For a linear cost function $TC = a + bq$ Show that the marginal cost is a constant.
6. Find the slope of the tangent line to the curve $y = x^3 - x$ at $x = 2$.
7. Give an example of a function that is continuous but not differential at a point.
8. Obtain a relation between the variables x and y if it is given that $x = t^2$ and $y = 2t$.
9. Write down the derivative of $\log_a x$.
10. What is the value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$?

(10 × 1 = 10 Marks)

SECTION – II

(Answer any eight questions. These questions carry 2 marks each).

11. Find the formula for the inverse of $f(x) = \sqrt{3x - 2}$.
12. Draw the graph of $xy = 5$.
13. If $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$ where n is positive integer. Find the value of n .
14. For the demand function $x = \frac{90}{p+5} - 6$, at what price does the demand tend to vanish?
15. For the demand function $q = 25 - 4p + p^2$, show that the elasticity of demand for $p = 5$ is unitary.
16. Discuss the continuity of the function $f(x) = \sqrt{9 - x^2}$.

17. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{x^2 + 1}$.
18. If f and g are continuous functions such that $\lim_{x \rightarrow 2} [f(x) + 4g(x)] = 13$ and $f(2) = 1$.
Find
(a) $g(2)$
(b) $\lim_{x \rightarrow 2} g(x)$.
19. For what values of x is there a discontinuity in the graph of $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$?
20. If the demand law is given by $p = \frac{a}{x} - c$. Show that the total revenue decreases as output increases.
21. Differentiate the function $\frac{x^2 - 1}{x^2 + 1}$ with respect to x .
22. Find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$.
23. Find the slope of the tangent line to the curve $x + y + xy = 3$ at the point $(1, 1)$.
24. If x and y satisfy the relation $x^2 + y^2 = 1$. Show that $\frac{dy}{dx} = -\frac{x}{y}$.
25. If the total cost function is $\pi = ax^2 + bx + c$, find the marginal cost.
26. What is law of demand?

(8 × 2 = 16 Marks)

SECTION – III

(Answer any six questions. These questions carry 4 marks each).

27. Find the value of k so that $\begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$ is a continuous function.

28. For the curve $q = 36 - 2p$, what can you conclude about the slopes of the MR function and the AR function?

29. Find all values of x for which the tangent line to the curve $y = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 5$.

30. If $f(x)$ is a single valued function of x , express the derivatives of $\sqrt{f(x)}$ and its reciprocal in terms of the derivative of $f(x)$.

31. Find $\frac{dy}{dx}$ when

(a) $4x^2 - 2y^2 = 9$

(b) $y = \frac{1}{\sqrt{x+1}}$

32. If $y = 3x^4 - 2x^3 + x^2 - 4x + 2$, find $\frac{dy}{dx}$.

33. Find the function inverse to $y = \frac{2x+1}{x-1}$ and show that it is single valued.

Represent it graphically.

34. Find a relation between marginal revenue MR and elasticity of demand e_d .

35. If $\lim_{x \rightarrow +\infty} f(x) = 2$ and $\lim_{x \rightarrow +\infty} g(x) = -3$, find whether the limit $\lim_{x \rightarrow +\infty} \frac{2f(x) + 3g(x)}{3f(x) + 2g(x)}$ exist or not.
36. Show that the marginal revenue can always be expressed as $p + x \frac{dp}{dx}$. Deduce that the gradient of the demand curve is numerically equal to $\frac{p}{x}$ at the output where marginal revenue is zero.
37. If $f(x) = \frac{4x^2 - x}{2x^3 - 5}$, find $\lim_{x \rightarrow -\infty} f(x)$.
38. If $f(x) = 4 + \frac{1}{1 + \frac{1}{1-x}}$, $0 < x < 1$, find $\lim_{x \rightarrow 1} f(x)$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer any two questions. These questions carry 15 marks each).

39. (a) If $f(x) = x^2$ and $g(x) = \sin x$, find $(f \circ g)(x)$ and $(g \circ f)(x)$
- (b) Find the domain and range of $f(x) = \frac{1+x}{1-x}$.
- (c) If $f(x) = \frac{x^2 + 3x - 2}{x^2 + 2x + 4}$, express $f(2a)$ in terms of a
40. (a) Given that the function $f(x) = \begin{cases} 5-x & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$
- (i) Draw the graph of the function
- (ii) Identify the discontinuity of the function in the graph
- (iii) Find $\lim_{x \rightarrow 4} (5-x)$ and show that the value of the limit is not equal to the value of the function at $x=4$. What do you conclude?
- (b) From the relation $xy + 2x + y - 1 = 0$, find the limit of y as $x \rightarrow 1$ and the limit of x as $y \rightarrow 1$.

41. (a) Explain the concepts of total revenue curve, average and marginal revenue curves
- (b) The total revenue received from sale of x units of a product is given by $R(x) = 12x + 2x^2 + 6$. Find (i) the average revenue (ii) the marginal revenue (iii) marginal revenue at $x=50$ (iv) the actual revenue from selling 51st item.
42. (a) Use differential calculus to explain the concept of marginal revenue
- (b) The demand function of a monopolists is given by $p = 1500 - 2x - x^2$. Find the total revenue, average revenue and marginal revenue functions. Also find the marginal revenue at $x=20$
43. Let π be the total cost and q the quantity of output produced and also that the cost function is given by $\pi = a + bq + cq^2$ where a , b , and c are constants. Find the
- (a) Marginal Cost (MC)
- (b) Average Cost (AC)
- (c) Relation between average and marginal cost
44. Find $\frac{dy}{dx}$ if
- (a) $y = \frac{e^{2x} \ln x}{x^2}$
- (b) $y = \sqrt{\frac{1+x}{1-x}}$
- (c) $x^2 + y^2 + 2x + 4y + 5 = 0$.

(2 × 15 = 30 Marks)