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N – 3967

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Core Course

Mathematics

MM 1141 : METHODS OF MATHEMATICS

(2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer all question compulsory. Each question carries 1 mark.

1. Find an equation for the local linear approximation to $y=5-x^2$ at $x_0=2$.
2. If f has an inflection point at $x=x_0$, then $f''(x_0) = \underline{\hspace{2cm}}$
3. State the extreme value Theorem.
4. The function $f(x)=|x| - 1$ has how many horizontal tangent to the graph of f over $(-1,1)$.
5. If f is differentiable and $f'(x) \neq 0$ an (a, b) , then the equation $f(x) = 0$.
6. Find the velocity and speed of the function $s(t)=t^3 - 6t^2$.

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7. A cylindrical shell is enclosed by two concentric right circular cylinders. The volume of that cylindrical shell is _____
8. The lateral area of the frustum with slant height $\sqrt{10}$ and base radii $r_1=1$ and $r_2=2$ is _____
9. The volume of the torus generated by revolving a circular region of radius b about a line at a distance a from the centre of the circle is _____
10. Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1,4]$ is revolved about the x-axis.

(10 × 1 = 10 Marks)

SECTION – II

Answer any eight questions. Each question carries 2 marks.

11. Express the derivative with respect to x of $y=x^2$ in differential form at $x=1$.
12. Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ using L' Hospital's rule.
13. Find the inflection points, if any, of $f(x)=x^4$.
14. Find all critical points of $f(x)=3x^{\frac{5}{3}}-15x^{\frac{2}{3}}$.
15. Find $\frac{dy}{dx}$ for $y=\frac{x^2-1}{x^3}$.
16. Find the absolute extrema of $f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}$ in the interval $[-1, 1]$.
17. Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1, 4]$ is resolved about the x-axis.

18. A spring exerts a force of 5 N when stretched 1 m beyond its natural length. Find the spring constant k .
19. Find the fluid pressure and force on the top of a flat circular plate of radius 2 m that is submerged horizontally in water at a depth of 6 m.
20. Prove that $\cosh^2 x - \sinh^2 x = 1$.
21. Find the value of $\int_{-2}^3 ((x+6) - x^2) dx$.
22. Define Hooke's law.

(8 × 2 = 16 Marks)

SECTION – III

Answer any six questions. Each question carries 4 marks.

23. The diameter of a polyurethane sphere is measured with percentage error within $\pm 0.4\%$. Estimate the percentage error in the calculated volume of the sphere.
24. Find $\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$.
25. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.
26. Explain the steps for solving Applied Maximum and Minimum Problems.
27. Find the absolute extrema, if any, of the function $f(x) = e^{(x^3 - 3x^2)}$ on the interval $(0, +\infty)$.
28. Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a .
29. A space probe of mass $m = 5.00 \times 10^4$ kg travels in deep space subjected only to the force of its own engine. Starting at a time when the speed of the probe is $v = 1.10 \times 10^4$ m/s, the engine is fired continuously over a distance of 2.50×10^6 m with a constant force of 4.00×10^5 N in the direction of motion. What is the final speed of the probe.

30. Explain Fluid Pressure.

31. A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If the total production cost for x units is $C(x) = 500,000 + 80x + 0.003x^2$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any two questions. Each question carries 15 marks.

32. Find a point on the curve $y = x^2$ that is closest to the point (18,0).

33. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

34. Suppose that the position function of a particle moving on a coordinate line is given by $s(t) = 2t^3 - 21t^2 + 60t + 3$.

35. Explain the surface area problem.

(2 × 15 = 30 Marks)