

Reg. No. :

Name :

Fifth Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Core Course VIII

MM 1544 – DIFFERENTIAL EQUATIONS

(2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions :

1. Write the standard equation of linear differential equation.
2. Write the Lipschitz condition.
3. Solve $dy + y dx = 0$.
4. For what values of the constant m will $y = e^{mx}$ be the solution of $y'' - 3y' - 10y = 0$.
5. Check whether $y^2 dy + x^2 dx$ exact or not.
6. Find the complementary function of $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^x \sin x$.
7. Define Wronskian
8. Write the standard form of Legendre's linear equation

9. Write the characteristic equation of $2 \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 3y = 0$.
10. Define basis of solutions of a homogeneous second order differential equation

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions

11. Find the order and degree of the ODE $\frac{d^3 y}{dx^3} + 2 \left(\frac{dy}{dx} \right)^{\frac{1}{2}} = 0$
12. Define partial differential equation. Give one example of it
13. Solve $\frac{dy}{dx} = xy + x$.
14. State the uniqueness theorem of first order differential equation.
15. Verify that $y = \frac{2}{x}$ is a solution of the differential equation $xy' = -y$, for all $x \neq 0$.
16. Show that a separable equation is also exact.
17. Check the exactness of $y' = 1 + y^2$.
18. Find the integrating factor of $y dx - x dy = 0$.
19. Find the general solution of $\frac{d^2 y}{dx^2} + 4y = 0$.
20. Find a differential equation whose solution is $\cos 3x$.
21. Find the complementary function of $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 3e^x$.
22. Write the basis of solution of the equation $\frac{d^2 y}{dx^2} + y = 0$
23. Write the standard form of Euler- Cauchy equation. Give one example of it.

24. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2 = 0$
25. Find a general solution of $x^2y' - 20y = 0$.
26. Find the Wronskian of e^x and e^{-x} .

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions

27. Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$
28. Solve $(x + 4)(y^2 + 1)dx + y(x^2 + 3x + 2)dy = 0$
29. Find the Orthogonal Trajectories of the family $cx^2 + y^2 = 1$
30. Solve the initial value problem $y' + y \tan x = \sin 2x, y(0) = 1$
31. Solve $x \frac{dy}{dx} + y = xy^{\frac{3}{2}}, y(1) = 4$.
32. Solve $(x^2 - 3y^2)dx + 2xy dy = 0$
33. By reducing the order, solve $(x^2 + 1)y'' - 2xy' + 2y = 0$, given x is one solution
34. Solve $\frac{d^2y}{dx^2} + y = \sin x$.
35. Find the general solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$
36. Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 16y = 0$.
37. Solve the logistic equation $y' = Ay - By^2$.
38. Solve $y'' + y = \operatorname{cosec} x$ using the method of variation of parameters.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions

39. (a) Solve $\left(\frac{3-y}{x^2}\right)dx + \left(\frac{y^2-2x}{xy^2}\right)dy = 0$, $y(-1) = 2$ by exactness.
- (b) Find an integrating factor and solve
 $(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$
40. (a) Solve the initial value problem $(ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0$, $y(0) = 6$
- (b) Find a basis of solutions of the differential equation $(x^2 - x)y'' - xy' + y = 0$.
41. (a) Check the exactness and solve $(2xy^2 + y)dx + (2y^3 - x)dy = 0$.
- (b) Solve the initial value problem $(y + \sqrt{x^2 + y^2})dx - xdy = 0$, $y(1) = 0$.
42. (a) Solve $x^2y'' - 2xy' + 2y = 0$, $y(1) = 1$, $y'(1) = 1$.
- (b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$.
43. (a) Solve $(D^2 + 2D + \frac{3}{4})y = 3e^x + \frac{9}{2}x$.
- (b) Solve $y''' - 3y'' + 2y' = 0$.
44. (a) Solve the initial value problem
 $y'' - 2y' - 3y = 2e^x - 10\sin x$, $y(0) = 2$, $y'(0) = 4$
- (b) Solve $(D^2 + 3D + 2)y = 5x^2$.

(2 × 15 = 30 Marks)